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ON HERMITE-HADAMARD TYPE INEQUALITIES FOR $s$–CONVEX MAPPINGS VIA FRACTIONAL INTEGRALS OF A FUNCTION WITH RESPECT TO ANOTHER FUNCTION

Abstract. In this paper, we obtain some Hermite-Hadamard type inequalities for $s$–convex function via fractional integrals with respect to another function which generalize the Riemann-Liouville fractional integrals and the Hadamard fractional integrals. The results presented here provide extensions of those given in earlier works.

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1. Introduction

Definition 1. The function $f : [a, b] \subset \mathbb{R} \to \mathbb{R}$ is said to be convex if the following inequality holds

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y)$$

for all $x, y \in [a, b]$ and $\lambda \in [0, 1]$. We say that $f$ is concave if $(-f)$ is convex.

Definition 2 ([4]). Let $s \in (0, 1]$. A function $f : [0, \infty) \to [0, \infty)$ is said to be $s$–convex (in the second sense), or that $f$ belongs to the class $K^2_s$, if

$$f(\lambda x + (1 - \lambda)y) \leq \lambda^s f(x) + (1 - \lambda)^s f(y)$$

for all $x, y \in [0, \infty)$ and $\lambda \in [0, 1]$.

An $s$–convex function was introduced in Breckner’s paper [4] and a number of properties and connections with $s$–convexity in the first sense were discussed in paper [13].
The inequalities discovered by C. Hermite and J. Hadamard for convex functions are very important in the literature (see, e.g., [17, p.137], [10]). These inequalities state that if \( f : I \to \mathbb{R} \) is a convex function on the interval \( I \) of real numbers and \( a, b \in I \) with \( a < b \), then

\[
(1) \quad f \left( \frac{a + b}{2} \right) \leq \frac{1}{b - a} \int_a^b f(x) \, dx \leq \frac{f(a) + f(b)}{2}.
\]

Both inequalities hold in the reversed direction if \( f \) is concave. We note that Hadamard’s inequality may be regarded as a refinement of the concept of convexity and it follows easily from Jensen’s inequality. Hadamard’s inequality for convex functions has received renewed attention in recent years and a remarkable variety of refinements and generalizations have been studied (see, for example, [1, 2, 10, 11, 17, 22, 23]).

In the following we present a brief synopsis of all necessary definitions and results that will be required. More details, one can consult [12, 15, 16, 18].

**Definition 3.** Let \( f \in L_1[a, b] \). The Riemann-Liouville fractional integrals \( J_{a+}^\alpha f \) and \( J_{b-}^\alpha f \) of order \( \alpha > 0 \) with \( a \geq 0 \) are defined by

\[
J_{a+}^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x - t)^{\alpha-1} f(t) \, dt, \quad x > a
\]

and

\[
J_{b-}^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_x^b (t - x)^{\alpha-1} f(t) \, dt, \quad x < b
\]

respectively. Here, \( \Gamma(\alpha) \) is the Gamma function and \( J_{a+}^0 f(x) = J_{b-}^0 f(x) = f(x) \).

It is remarkable that Sarikaya et al. [20] first gave the following interesting integral inequalities of Hermite-Hadamard type involving Riemann-Liouville fractional integrals.

**Theorem 1.** Let \( f : [a, b] \to \mathbb{R} \) be a positive function with \( 0 \leq a < b \) and \( f \in L_1[a, b] \). If \( f \) is a convex function on \([a, b]\), then the following inequalities for fractional integrals hold:

\[
(2) \quad f \left( \frac{a + b}{2} \right) \leq \frac{\Gamma(\alpha + 1)}{2(b - a)^\alpha} \left[ J_{a+}^\alpha f(b) + J_{b-}^\alpha f(a) \right] \leq \frac{f(a) + f(b)}{2}
\]

with \( \alpha > 0 \).

**Definition 4.** Let \( f \in L_1[a, b] \). The Hadamard fractional integrals \( J_{a+}^\alpha f \) and \( J_{b-}^\alpha f \) of order \( \alpha > 0 \) with \( a \geq 0 \) are defined by

\[
J_{a+}^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x \left( \ln \frac{x}{t} \right)^{\alpha-1} f(t) \, dt, \quad x > a
\]
On Hermite-Hadamard type inequalities . . .

and

\[ J_{b-}^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_x^b \left( \ln \frac{t}{x} \right)^{\alpha-1} f(t) dt, \quad x < b \]

respectively.

**Definition 5.** Let \( g : [a,b] \to \mathbb{R} \) be an increasing and positive monotone function on \((a,b)\), having a continuous derivative \( g'(x) \) on \((a,b)\). The left-sides \( I_{a+}^{\alpha,g} f(x) \) and right-sides \( I_{b-}^{\alpha,g} f(x) \) fractional integral of \( f \) with respect to the function \( g \) on \([a,b]\) of order \( \alpha < 0 \) are defined by

\[ I_{a+}^{\alpha,g} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x \frac{g'(t)f(t)}{[g(x)-g(t)]^{1-\alpha}} dt, \quad x > a \]

and

\[ I_{b-}^{\alpha,g} f(x) = \frac{1}{\Gamma(\alpha)} \int_x^b \frac{g'(t)f(t)}{[g(t)-g(x)]^{1-\alpha}} dt, \quad x < b \]

respectively.

In [14], Jleli and Samet gave the following equality:

**Lemma 1.** Let \( \alpha > 0 \) and let \( \Xi_{\alpha,g} : [0,1] \to \mathbb{R} \) be a function defined by

\[ \Xi_{\alpha,g}(t) = \left[ g(ta + (1-t)b) - g(a) \right]^\alpha - \left[ g(tb + (1-t)a) - g(a) \right]^\alpha + \left[ g(b) - g(tb + (1-t)a) \right]^\alpha - \left[ g(b) - g(ta + (1-t)b) \right]^\alpha. \]

If \( f \in C^1(I^\circ) \), then

\[ \frac{f(a) + f(b)}{2} = \frac{\Gamma(\alpha + 1)}{4 [g(b) - g(a)]^{\alpha}} \left( I_{a+}^{\alpha,g} F(b) + I_{b-}^{\alpha,g} F(a) \right) \]

\[ = \frac{b-a}{4 [g(b) - g(a)]^{\alpha}} \int_0^1 \Xi_{\alpha,g}(t) f'(ta + (1-t)b) dt. \]

For some recent results connected with fractional integral inequalities, see [3], [5]-[9], [19], [21], [24]-[27].

The aim of this paper is to establish generalized Hermite-Hadamard type integral inequalities for \( s \)-convex function involving fractional integrals with respect to another function. The results presented in this paper provide extensions of those given in earlier works.

### 2. Main results

Firstly, let us start with some notations given in [14]. Let \( f : I^\circ \to \mathbb{R} \) be a function such that \( a, b \in I^\circ \) and \( 0 < a < b < \infty \). We suppose that
$f \in L^\infty(a,b)$ in such a way that $I_{a+g}^\alpha f(x)$ and $I_{b-g}^\alpha f(x)$ are well defined. We define the function

$$\tilde{f}(x) = f(a + b - x), \quad x \in [a, b]$$

$$F(x) = f(x) + \tilde{f}(x), \quad x \in [a, b].$$

Now we shall present the following notations:

$$H_1(\alpha, s; g) = \int_0^1 \frac{[t^s + (1-t)^s] g'((1-t)a + tb)}{[g(b) - g((1-t)a + tb)]^{1-\alpha}} dt,$$

$$H_2(\alpha, s; g) = \int_0^1 \frac{[t^s + (1-t)^s] g'((1-t)a + tb)}{[g((1-t)a + tb) - g(a)]^{1-\alpha}} dt.$$

For $g(t) = t$, we have

$$H_1(\alpha, s; g) = H_2(\alpha, s; g) = (b - a)^{\alpha-1} \left[ \frac{1}{\alpha + s} + \beta(\alpha, s + 1) \right]$$

where $\beta(x, y)$ is the Beta function.

For $\alpha > 0$ and $s \in (0, 1]$, we give the following operator

$$L_\alpha^g(x, y) = \int_a^{\frac{a+b}{2}} |x - u|^s |g(y) - g(u)|^\alpha du - \int_{\frac{a+b}{2}}^b |x - u|^s |g(y) - g(u)|^\alpha du$$

for $x, y \in [a, b]$. Particularly, for $g(t) = t$, we have

$$L_\alpha^g(b, b) = -L_\alpha^g(a, a) = (b - a)^{\alpha+s+1} \frac{2^{\alpha+s} - 1}{2^{\alpha+s}(\alpha + s + 1)}$$

and

$$L_\alpha^g(a, b) = -L_\alpha^g(b, a)$$

$$= (b - a)^{\alpha+s+1} \left[ \beta\left(\frac{1}{2}; s + 1, \alpha + 1 \right) - \beta\left(\frac{1}{2}; \alpha + 1, s + 1 \right) \right]$$

where $\beta(z; x, y)$ is the incomplete Beta function.

**Theorem 2.** Let $g : [a, b] \to \mathbb{R}$ be an increasing and positive monotone function on $(a, b)$, having a continuous derivative $g'(x)$ on $(a, b)$ and let $\alpha > 0$. If $f$ is a $s$-convex in the second sense on $[a, b]$ for some fixed $s \in (0, 1]$,
then the following Hermite-Hadamard type inequality for fractional integrals hold:

\[
2^{s-1} f \left( \frac{a + b}{2} \right) \leq \frac{\Gamma(\alpha + 1)}{2 [g(b) - g(a)]^\alpha} \left[ \frac{I_{a+;g}^\alpha F(b) + I_{b-;g}^\alpha F(a)}{2} \right]
\]

\[
\leq \left[ \frac{f(a) + f(b)}{2} \right] \times \frac{\alpha (b - a)}{|g(b) - g(a)|^\alpha} \left[ \frac{H_1(\alpha, s; g) + H_2(\alpha, s; g)}{2} \right].
\]

**Proof.** Since \( f \) is an \( s \)-convex mapping in the second sense on \([a, b]\), we have

\[
f \left( \frac{x + y}{2} \right) \leq \frac{f(x) + f(y)}{2^s}
\]

for \( x, y \in [a, b] \). Now, for \( t \in [0, 1] \), let \( x = ta + (1 - t)b \) and \( y = (1 - t)a + tb \). Then we have

\[
2^s f \left( \frac{a + b}{2} \right) \leq f \left( ta + (1 - t)b \right) + f \left( (1 - t)a + tb \right).
\]

Multiplying both sides of (5) by

\[
\frac{b - a}{\Gamma(\alpha)} \frac{g' ((1 - t)a + tb)}{[g(b) - g ((1 - t)a + tb)]^{1-\alpha}}
\]

and integrating the resulting inequality with respect to \( t \) over \((0, 1)\), we get

\[
\frac{2^s (b - a)}{\Gamma(\alpha)} f \left( \frac{a + b}{2} \right) \frac{1}{0} \int \frac{g' ((1 - t)a + tb)}{[g(b) - g ((1 - t)a + tb)]^{1-\alpha}} dt
\]

\[
\leq \frac{b - a}{\Gamma(\alpha)} \frac{1}{0} \int f \left( ta + (1 - t)b \right) \frac{g' ((1 - t)a + tb)}{[g(b) - g ((1 - t)a + tb)]^{1-\alpha}} dt
\]

\[
+ \frac{b - a}{\Gamma(\alpha)} \frac{1}{0} \int f \left( (1 - t)a + tb \right) \frac{g' ((1 - t)a + tb)}{[g(b) - g ((1 - t)a + tb)]^{1-\alpha}} dt.
\]

Using the change of variable \( \tau = (1 - t)a + tb \), we have

\[
\frac{2^s}{\Gamma(\alpha)} f \left( \frac{a + b}{2} \right) \frac{[g(b) - g(a)]^\alpha}{\alpha} \leq I_{a+;g}^{\alpha} \tilde{f}(b) + I_{a+;g}^{\alpha} f(b),
\]
i.e.

\[
\frac{2^s}{\Gamma(\alpha + 1)} f \left( \frac{a + b}{2} \right) \left[ g(b) - g(a) \right]^\alpha \leq I_{a^+,g}^{\alpha} F(b).
\]

Similarly, multiplying both sides of (5) by

\[
b - a \frac{g'((1 - t)a + tb)}{\Gamma(\alpha) \left[ g((1 - t)a + tb) - g(a) \right]^{1-\alpha}}
\]

and integrating the resulting inequality with respect to \( t \) over \((0, 1)\), we obtain

\[
\frac{2^s}{\Gamma(\alpha + 1)} f \left( \frac{a + b}{2} \right) \left[ g(b) - g(a) \right]^\alpha \leq I_{b^-,g}^{\alpha} F(a).
\]

Summing the inequalities (6) and (7), we get

\[
2^{s-1} f \left( \frac{a + b}{2} \right) \leq \frac{\Gamma(\alpha + 1)}{2 \left[ g(b) - g(a) \right]^\alpha} \left[ I_{a^+,g}^{\alpha} F(b) + I_{b^-,g}^{\alpha} F(a) \right].
\]

This completes the proof of first inequality in (3).

For the proof of the second inequality in (3), since \( f \) is \( s \)-convex in the second sense, we have

\[
f (ta + (1 - t)b) \leq t^s f(a) + (1 - t)^s f(b)
\]

and

\[
f ((1 - t)a + tb) \leq (1 - t)^s f(a) + t^s f(b).
\]

By adding these inequalities, we have

\[
f (ta + (1 - t)b) + f ((1 - t)a + tb) \leq \left[ t^s + (1 - t)^s \right] \left[ f(a) + f(b) \right].
\]

Multiplying both sides of (8) by

\[
b - a \frac{g'((1 - t)a + tb)}{\Gamma(\alpha) \left[ g((1 - t)a + tb) - g(a) \right]^{1-\alpha}}
\]

and integrating the resulting inequality with respect to \( t \) over \((0, 1)\), we have

\[
\frac{b - a}{\Gamma(\alpha)} \int_0^1 f (ta + (1 - t)b) g'((1 - t)a + tb) \frac{1}{\left[ g(b) - g((1 - t)a + tb) \right]^{1-\alpha}} dt
\]

\[
+ \frac{b - a}{\Gamma(\alpha)} \int_0^1 f ((1 - t)a + tb) g'((1 - t)a + tb) \frac{1}{\left[ g(b) - g((1 - t)a + tb) \right]^{1-\alpha}} dt
\]

\[
\leq \left[ f(a) + f(b) \right] \frac{b - a}{\Gamma(\alpha)} \int_0^1 \frac{t^s + (1 - t)^s}{\left[ g(b) - g((1 - t)a + tb) \right]^{1-\alpha}} dt.
\]
Then, we get
\[
I_{a^+;g}^\alpha \tilde{f}(b) + I_{a^+;g}^\alpha f(b) \leq \left[ f(a) + f(b) \right] \frac{b-a}{\Gamma(\alpha)} H_1(\alpha, s; g),
\]
that is,
\[
I_{a^+;g}^\alpha F(b) \leq \left[ f(a) + f(b) \right] \frac{b-a}{\Gamma(\alpha)} H_1(\alpha, s; g).
\]
(9)

Similarly, multiplying both sides of (8) by
\[
\frac{b-a}{\Gamma(\alpha)} \frac{g'((1-t)a+tb)}{[g((1-t)a+tb) - g(a)]^{1-\alpha}}
\]
and integrating the resulting inequality with respect to \( t \) over \((0,1)\), we get
\[
I_{b^-;g}^\alpha F(a) \leq \left[ f(a) + f(b) \right] \frac{b-a}{\Gamma(\alpha)} H_2(\alpha, s; g).
\]
(10)

By adding the inequalities (9) and (10), we have
\[
\frac{\Gamma(\alpha + 1)}{2 [g(b) - g(a)]^{\alpha}} \left[ \frac{I_{b^-;g}^\alpha F(a) + I_{a^+;g}^\alpha F(b)}{2} \right] \leq \left[ \frac{f(a) + f(b)}{2} \right] \frac{\alpha (b-a)}{[g(b) - g(a)]^{\alpha}} \left[ \frac{H_1(\alpha, s; g) + H_2(\alpha, s; g)}{2} \right],
\]
which completes the proof.

\[\blacksquare\]

**Remark 1.** If we put \( s = 1 \) in (3), then we obtain Theorem 2.1 in [14].

**Remark 2.** If we choose \( g(t) = t \), then we obtain the following inequality
\[
2^{s-1} f \left( \frac{a + b}{2} \right) \leq \Gamma(\alpha + 1) \frac{J_a^\alpha f(b) + J_b^\alpha f(a)}{(b-a)^\alpha} \left[ \frac{H_1(\alpha, s; g) + H_2(\alpha, s; g)}{2} \right],
\]
which was proved by Set et al. in [25].

**Corollary 1.** Under assumption of Theorem 2 with \( g(t) = \ln t \), we have the following inequality
\[
2^{s-1} f \left( \frac{a + b}{2} \right) \leq \Gamma(\alpha + 1) \frac{J_a^\alpha F(b) + J_b^\alpha F(a)}{(\ln \frac{b}{a})^{\alpha}} \left[ \frac{H_1(\alpha, s; \ln) + H_2(\alpha, s; \ln)}{2} \right].
\]
Theorem 3. Let \( g \) be as the above. If \( f \in C_1(I^c) \) and \( |f'| \) is an \( s \)-convex in the second sense on \([a, b]\) for some fixed \( s \in (0, 1] \), then we have the inequality

\[
\left| \frac{f(a) + f(b)}{2} - \frac{\Gamma(\alpha + 1)}{4[g(b) - g(a)]^\alpha} \left( I_{a+g}^\alpha F(b) + I_{b-g}^\alpha F(a) \right) \right| \\
\leq \frac{I_{g,s}^\alpha(a, b)}{4[g(b) - g(a)]^\alpha} \left[ |f'(a)| + |f'(b)| \right],
\]

where

\[
I_{g,s}^\alpha(a, b) = L_{g,s}^\alpha(b, b) + L_{g,s}^\alpha(a, b) - L_{g,s}^\alpha(b, a) - L_{g,s}^\alpha(a, a).
\]

Proof. Taking modulus in Lemma 1, we have

\[
\left| \frac{f(a) + f(b)}{2} - \frac{\Gamma(\alpha + 1)}{4[g(b) - g(a)]^\alpha} \left( I_{a+g}^\alpha F(b) + I_{b-g}^\alpha F(a) \right) \right| \\
\leq \frac{b - a}{4[g(b) - g(a)]^\alpha} \int_0^1 |\Xi_{\alpha,g}(t)| |f'(ta + (1 - t)b)| \, dt.
\]

Since \( |f'| \) is an \( s \)-convex in the second sense on \([a, b]\), we get

\[
|f'(ta + (1 - t)b)| \leq t^s |f'(a)| + (1 - t)^s |f'(b)|.
\]

Hence,

\[
(11) \left| \frac{f(a) + f(b)}{2} - \frac{\Gamma(\alpha + 1)}{4[g(b) - g(a)]^\alpha} \left( I_{a+g}^\alpha F(b) + I_{b-g}^\alpha F(a) \right) \right| \\
\leq \frac{b - a}{4[g(b) - g(a)]^\alpha} \\
\times \left[ |f'(a)| \int_0^1 t^s |\Xi_{\alpha,g}(t)| \, dt + |f'(b)| \int_0^1 (1 - t)^s |\Xi_{\alpha,g}(t)| \, dt \right].
\]

Here, we have

\[
\int_0^1 t^s |\Xi_{\alpha,g}(t)| \, dt = \frac{1}{(b - a)^{s+1}} \int_a^b (b - u)^s |\varphi(u)| \, du
\]

where

\[
\varphi(u) = [g(u) - g(a)]^\alpha - [g(a + b - u) - g(a)]^\alpha \\
+ [g(b) - g(a + b - u)]^\alpha - [g(b) - g(u)]^\alpha.
\]
Since \( g \) is an increasing function, \( \varphi \) is a non-decreasing function on \([a, b]\).

Additionally,
\[
\varphi(a) = -2 [g(b) - g(a)]^\alpha < 0
\]

and
\[
\varphi\left(\frac{a + b}{2}\right) = 0.
\]

Consequently, we get
\[
\begin{cases}
\varphi(u) \leq 0, & \text{if } a \leq u \leq \frac{a+b}{2}, \\
\varphi(u) > 0, & \text{if } \frac{a+b}{2} < u \leq b.
\end{cases}
\]

Therefore, we have
\[
\int_a^b (b - u)^s |\varphi(u)| \, dt = I_1 + I_2 + I_3 + I_4
\]

where

\[
I_1 = \int_a^{\frac{a+b}{2}} (b - u)^s [g(b) - g(u)]^\alpha \, du
\]

\[
- \int_{\frac{a+b}{2}}^b (b - u)^s [g(b) - g(u)]^\alpha \, du = L_g^{\alpha,s}(b, b),
\]

\[
I_2 = -\int_a^{\frac{a+b}{2}} (b - u)^s [g(u) - g(a)]^\alpha \, du
\]

\[
+ \int_{\frac{a+b}{2}}^b (b - u)^s [g(u) - g(a)]^\alpha \, du = -L_g^{\alpha,s}(b, a),
\]

\[
I_3 = \int_a^{\frac{a+b}{2}} (b - u)^s [g(a + b - u) - g(a)]^\alpha \, du
\]

\[
- \int_{\frac{a+b}{2}}^b (b - u)^s [g(a + b - u) - g(a)]^\alpha \, du = -L_g^{\alpha,s}(a, a),
\]
and
\[
I_4 = - \int_a^b (b - u)^s \left[ g(b) - g(a + b - u) \right] \frac{1}{\alpha} du \\
+ \int_{\frac{a + b}{2}}^b (b - u)^s \left[ g(b) - g(a + b - u) \right] \frac{1}{\alpha} du = L^{\alpha,s}_g(a, b).
\]

Thus, from the previous equalities it follows that
\[
(12) \quad \int_0^1 t^s |\Xi_{\alpha,g}(t)| \, dt = \frac{L^{\alpha,s}_g(b, b) + L^{\alpha,s}_g(a, b) - L^{\alpha,s}_g(b, a) - L^{\alpha,s}_g(a, a)}{(b - a)^{s+1}}.
\]

Similarly, it is clear that
\[
(13) \quad \int_0^1 (1-t)^s |\Xi_{\alpha,g}(t)| \, dt = \frac{L^{\alpha,s}_g(b, b) + L^{\alpha,s}_g(a, b) - L^{\alpha,s}_g(b, a) - L^{\alpha,s}_g(a, a)}{(b - a)^{s+1}}.
\]

If we put equality (12) and (13) in (11), we obtain the desired result.

\[ \blacksquare \]

**Remark 3.** If we put \( s = 1 \) in (3), then we obtain Theorem 2.5 in [14].

**Remark 4.** If we choose \( g(t) = t \), then we obtain the following inequality
\[
\left| \frac{f(a) + f(b)}{2} - \frac{\Gamma(\alpha + 1)}{2 (b - a)^\alpha} \left[ J_\alpha^a f(b) + J_\alpha^b f(a) \right] \right| \\
\leq \frac{b - a}{2} \left\{ \beta \left( \frac{1}{2}; s + 1, \alpha + 1 \right) - \beta \left( \frac{1}{2}; \alpha + 1, s + 1 \right) \\
+ \frac{2^{\alpha+s} - 1}{2^{\alpha+s}(\alpha + s + 1)} \right\} \left[ |f'(a)| + |f'(b)| \right].
\]

This inequality was given by Set et al. in [25, Theorem 4 (for \( q = 1 \)].

**Corollary 2.** Under assumption of Theorem 2 with \( g(t) = \ln t \), we have the following inequality
\[
\left| \frac{f(a) + f(b)}{2} - \frac{\Gamma(\alpha + 1)}{4 (\ln \frac{b}{a})^\alpha} \left( J_\alpha^a F(b) + J_\alpha^b F(a) \right) \right| \\
\leq \frac{I_{\ln}^{\alpha,s}(a, b)}{4 (\ln \frac{b}{a})^\alpha} \left[ |f'(a)| + |f'(b)| \right],
\]
\[ \text{where} \]
\[
I_{\ln}^{\alpha,s}(a, b) = L_{\ln}^{\alpha,s}(b, b) + L_{\ln}^{\alpha,s}(a, b) - L_{\ln}^{\alpha,s}(b, a) - L_{\ln}^{\alpha,s}(a, a).
\]
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